Current-sheet formation in incompressible electron magnetohydrodynamics

V. P. Ruban*
L. D. Landau Institute for Theoretical Physics, 2 Kosygin Street, 117334 Moscow, Russia
and Optics and Fluid Dynamics Department, Risø National Laboratory, DK-4000 Roskilde, Denmark
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The nonlinear dynamics of axisymmetric, as well as helical, frozen-in vortex structures is investigated by the Hamiltonian method in the framework of ideal incompressible electron magnetohydrodynamics. For description of current-sheet formation from a smooth initial magnetic field, local and nonlocal nonlinear approximations are introduced and partially analyzed that are generalizations of the previously known exactly solvable local model neglecting electron inertia.

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It is a well known fact that current sheets play exclusively important role in plasma dynamics (see, e.g., [1–4] and references therein). However, analytical study of current-sheets formation and their dissipative dynamics is a very difficult problem in the framework of usually used nonlinear (and also nonlocal in the incompressible limit) equations of motion of plasmas. That concerns the usual magnetohydrodynamics (MHD), the electron magnetohydrodynamics (EMHD), as well as the multifluid models of plasmas. So, up to this day we do not have a mathematically clear answer on the question, whether the current density will become singular in a finite time or its growth can be only exponential in these systems. Numerical simulations remain to be the main tool for obtaining quantitative results [1–7]. Therefore, an important role for theoretical understanding of current-sheets dynamics can be played by local nonlinear approximations, that sometimes have exact solutions describing formation of singularities. An example of such relatively simple, approximate differential equation for the magnetic field $\mathbf{B}(\mathbf{r},t)$ is (see, e.g., [8] for derivation and explanation)

$$\mathbf{B}_t = -\frac{c}{4\pi e}\,\mathrm{curl}[(\mathrm{curl}\,\mathbf{B}/n)\times\mathbf{B}]. \tag{1}$$

This equation describes the motion of magnetic structures in EMHD on length scales much larger than the inertial electron skin depth, while the main part of the energy is concentrated in the magnetic field, with the kinetic energy of the electron fluid motion being much smaller. The equation (1) has been extensively exploited, for instance, to study fast penetration of magnetic field into plasmas due to the Hall effect [9,10], as well as rapid dissipation of magnetic fields in laboratory and astrophysical conditions [11]. The interest to this equation is explained, in particular, by the fact that axisymmetric configurations with $\mathbf{B}\|\mathbf{e}_\varphi$ have been found exactly solvable (see [8–11]). In this geometry, the equation of motion is reduced to the well known one-dimensional (1D) Hopf equation, that should be solved independently for each value of the radial coordinate. The mechanism of singularity formation in these solutions is connected simply with breaking in a finite time of the magnetic field profile. The magnetic field itself does not become infinite, but its curl tends to the infinity at some point of the axial cross section. Inclusion of dissipative terms into the equation stops the breaking, but instead of multivalued profile, a shock forms, the length of which increases with time. The shock is a cross section of a current sheet.

The main purpose of present work is to extend the analysis of such axisymmetric flows by consideration of additional nonlinear effects caused by electron inertia. They either play role of small corrections for long-scale flows or, when the shock becomes narrow, change drastically the dynamical behavior by smoothing the transport velocity field. This situation is quite different in comparison with the self-similar EMHD solutions discussed in Ref. [12]. Also, the flows with other geometrical symmetry are considered below in the approximation (1), when all the frozen-in magnetic lines have helical shapes with a same spatial period along z direction. In this case contours corresponding to different values of the axial component of the magnetic field rotate in a perpendicular plane with different angular velocities, thus producing the shock.

Incompressible two-fluid model. Before the main consideration, it is useful to recall the place of EMHD among different hydrodynamical plasma models [8]. If there are only two kinds of particles in the plasma—negatively charged electrons with the mass m and positively charged ions with the mass M, then the most general is the two-fluid model, which contains MHD, EMHD, and Hall MHD as special cases. Let the equilibrium concentration of particles of each sort be equal to n. If the temperature of the system is sufficiently large, $nT \gg \mathbf{B}^2$, then for slow vortical flows one can neglect deviations of the concentrations from n (the quasineutrality condition), and believe the velocity fields divergence-free in homogeneous case $(\nabla\cdot\mathbf{v}^{\pm})=0$.

Temporarily, we will not take into account dissipative processes. Thus, application of the canonical formalism becomes possible [13–15], which makes the analysis more compact. With appropriate choice for the length scales $[\sim d_+ = (Mc^2/4\pi e^2 n)^{1/2}]$ and for the mass scales $(\sim M)$, the Lagrangian functional of the incompressible two-fluid model, in the absence of an external magnetic field, takes the form

$$\mathcal{L}_\mu\{\mathbf{v}^+,\mathbf{v}^-\} = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\frac{|\mathbf{v}^+_{\mathbf{k}}|^2}{2} + \mu\frac{|\mathbf{v}^-_{\mathbf{k}}|^2}{2} + \frac{|\mathbf{v}^+_{\mathbf{k}} - \mathbf{v}^-_{\mathbf{k}}|^2}{2k^2}\right]. \tag{2}$$

*Electronic address: ruban@itp.ac.ru

Here $\mu=m/M$ is the only dimensionless parameter remaining in the system. For the electron-positron plasma $\mu=1$, for the hydrogen plasma $\mu\approx 1/2000\ll 1$. Below we consider the latter case. The first two terms in the expression (2) give the kinetic energy of the ion and electron fluids, while the third term is the energy of the magnetic field created by the flows of electrically charged fluids. The conditions of incompressibility are assumed, $(\mathbf{v}^{\pm}_{\mathbf{k}}\cdot\mathbf{k})=0$.

It is important that the variation of the action functional $S=\int\mathcal{L}_\mu dt$, which is necessary for constituting the equations of motion, should not be performed with respect to the variations $\delta\mathbf{v}^{\pm}(\mathbf{r},t)$, but with respect to the variations $\delta\mathbf{x}^{+}(\mathbf{a},t)$ and $\delta\mathbf{x}^{-}(\mathbf{c},t)$, where $\mathbf{x}^{+}(\mathbf{a},t)$ and $\mathbf{x}^{-}(\mathbf{c},t)$ are incompressible Lagrangian mappings describing the motion of points of the ion and electron fluids, labeled by the labels **a** and **c**. The corresponding mathematical technique is explained, for instance, in Refs. [16]. The equations of motion of the two-fluid incompressible system have the following structure:

$$\frac{\partial}{\partial t}\frac{\delta\mathcal{L}_\mu}{\delta\mathbf{v}^{\pm}(\mathbf{r})}=(1-\nabla\Delta^{-1}\nabla)\left[\mathbf{v}^{\pm}(\mathbf{r})\times\mathrm{curl}\frac{\delta\mathcal{L}_\mu}{\delta\mathbf{v}^{\pm}(\mathbf{r})}\right], \quad (3)$$

where the operator in the parentheses on the rhs is the projector onto the functional space of divergence-free 3D vector fields [13,16]. The two vector fields $\mathbf{p}^{\pm}(\mathbf{r})\equiv\delta\mathcal{L}_\mu/\delta\mathbf{v}^{\pm}(\mathbf{r})$ are the canonical momenta by definition. In the Fourier representation they are given by the expressions

$$\mathbf{p}^{+}_{\mathbf{k}}=\delta\mathcal{L}_\mu/\delta\mathbf{v}^{+}_{-\mathbf{k}}=[1+(1/k^2)]\mathbf{v}^{+}_{\mathbf{k}}-\frac{\mathbf{v}^{-}_{\mathbf{k}}}{k^2}, \quad (4)$$

$$\mathbf{p}^{-}_{\mathbf{k}}=\delta\mathcal{L}_\mu/\delta\mathbf{v}^{-}_{-\mathbf{k}}=[\mu+(1/k^2)]\mathbf{v}^{-}_{\mathbf{k}}-\frac{\mathbf{v}^{+}_{\mathbf{k}}}{k^2}. \quad (5)$$

Below, we will need the reversal relations for the velocities through the momenta,

$$\mathbf{v}^{+}_{\mathbf{k}}=\frac{(\mu k^2+1)\mathbf{p}^{+}_{\mathbf{k}}+\mathbf{p}^{-}_{\mathbf{k}}}{\mu k^2+1+\mu},\quad \mathbf{v}^{-}_{\mathbf{k}}=\frac{(k^2+1)\mathbf{p}^{-}_{\mathbf{k}}+\mathbf{p}^{+}_{\mathbf{k}}}{\mu k^2+1+\mu}. \quad (6)$$

It is possible to reformulate the equations (3) as equations for frozen-in vortices,

$$\mathbf{\Omega}^{\pm}_t(\mathbf{r})=\mathrm{curl}\left[\mathrm{curl}\frac{\delta\mathcal{H}_\mu}{\delta\mathbf{\Omega}^{\pm}(\mathbf{r})}\times\mathbf{\Omega}^{\pm}(\mathbf{r})\right], \quad (7)$$

where the canonical vorticity fields are defined as the curls of the canonical momenta, $\mathbf{\Omega}^{\pm}(\mathbf{r},t)\equiv\mathrm{curl}\,\mathbf{p}^{\pm}(\mathbf{r},t)$, and also the Hamiltonian functional of the system is calculated,

$$\mathcal{H}_\mu\{\mathbf{\Omega}^+,\mathbf{\Omega}^-\}\equiv\int\{(\mathbf{p}^+\cdot\mathbf{v}^+)+(\mathbf{p}^-\cdot\mathbf{v}^-)\}d\mathbf{r}-\mathcal{L}_\mu$$
$$=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left[\frac{(\mu k^2+1)|\mathbf{\Omega}^+_{\mathbf{k}}|^2+(k^2+1)|\mathbf{\Omega}^-_{\mathbf{k}}|^2+2(\mathbf{\Omega}^+_{\mathbf{k}}\cdot\mathbf{\Omega}^-_{-\mathbf{k}})}{2k^2(\mu k^2+1+\mu)}\right]. \quad (8)$$

It is clear that in the problem under consideration there are two separated dimensionless scales of inverse length, $k_+\sim 1$ and $k_-\sim 1/\lambda$, where $\lambda=\sqrt{\mu}$ is the electron inertial skin depth (normalized to d_+). Since $\lambda^2\ll 1$, we may write with very good accuracy $\mathcal{H}_\mu\{\mathbf{\Omega}^+,\mathbf{\Omega}^-\}\approx\mathcal{H}_\lambda\{\mathbf{\Omega}^+,\mathbf{\Omega}^-\}$, where

$$\mathcal{H}_\lambda\{\mathbf{\Omega}^+,\mathbf{\Omega}^-\}=\frac{1}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}[G_{++}(k)|\mathbf{\Omega}^+_{\mathbf{k}}|^2+G_{--}(k)|\mathbf{\Omega}^-_{\mathbf{k}}|^2+2G_{+-}(k)(\mathbf{\Omega}^+_{\mathbf{k}}\cdot\mathbf{\Omega}^-_{-\mathbf{k}})], \quad (9)$$

$$G_{++}(k)=\frac{1}{k^2},\quad G_{+-}(k)=\left(\frac{1}{k^2}-\frac{1}{k^2+\lambda^{-2}}\right),$$

$$G_{--}(k)=\left(\frac{1}{k^2}+\frac{1}{1+\lambda^2k^2}\right). \quad (10)$$

Depending on the typical spatial scale of the vortices, several dynamical regimes are possible in this system. The small and moderate wave number region, $k<\sim 1$, corresponds to the Hall MHD, and in the special limit $|\mathbf{\Omega}^+ +\mathbf{\Omega}^-|\ll|\mathbf{\Omega}^+|,|\mathbf{\Omega}^-|$, we have here the usual MHD. The region $1\ll k<\sim 1/\lambda$, under the extra condition $|\mathbf{\Omega}^+|\ll|\mathbf{\Omega}^-|$, corresponds to the EMHD [8]. For the flows with larger typical wave numbers, $k\gg 1/\lambda$, the magnetic effects become relatively insignificant, and the system (9) is broken into two weakly interacting subsystems, each of them being approximately described by the ordinary Eulerian hydrodynamics, since $G_{--}(k)\approx 1/\lambda^2k^2$, $G_{+-}(k)\approx 1/\lambda^2k^4\ll G_{++}(k),G_{--}(k)$ in this region.

Axisymmetric large-scale EMHD flows. Let us now consider the subset of solutions, for which the ion canonical vorticity is identically equal to zero, $\mathbf{\Omega}^+=0$, and the electron vorticity $\mathbf{\Omega}^-_{\mathbf{k}}$ is concentrated in the range $1\ll k\ll 1/\lambda$ of the wave numbers, where the Green's function $G_{--}(k)$ is almost flat $G_{--}(k)\approx 1$. Practically this corresponds to the condition $3<\sim k<\sim 20$. For EMHD model this is the long-scale region, where $\mathbf{\Omega}^-$ is proportional to the magnetic field in the leading order. It should be emphasized that with $\mathbf{\Omega}^+=0$ the velocity $\mathbf{v}^+$ of the ion component is not exactly zero, however, it is much smaller than the velocity $\mathbf{v}^-$ of the electron component, as it becomes clear from consideration of the Eqs. (6) with $\mathbf{p}^+=0$. In the main approximation, the Hamiltonian for the electron vorticity takes the very simple form

$$\mathcal{H}_\lambda\{\mathbf{0},\mathbf{\Omega}^-\}\approx\frac{1}{2}\int|\mathbf{\Omega}^-|^2 d\mathbf{r}, \tag{11}$$

in accordance with the fact that the energy of the system is concentrated mostly in the magnetic field. The corresponding equation of motion is local and essentially coincides with Eq. (1),

$$\mathbf{\Omega}_t^-=\mathrm{curl}\,[\mathrm{curl}\mathbf{\Omega}^-\times\mathbf{\Omega}^-]. \tag{12}$$

One of the remarkable properties of the equation (12) is that in the case of axisymmetric flows, when

$$\mathbf{\Omega}^-(\mathbf{r},t)=\omega^-(q,z,t)[\mathbf{e}_z\times\mathbf{r}], \tag{13}$$

where $q=(x^2+y^2)/2$, we have the exactly solvable Hopf equation for the function $\omega^-(q,z,t)$ [8],

$$\omega_t^-+2\omega^-\omega_z^-=0. \tag{14}$$

The solution of the equation (14) at $t>0$ is constructed from the initial function $\omega_0^-(q,z)$ by the shift of each level contour $\omega_0^-(q,z)=w$ along z axis on the value $2wt$, that makes possible breaking of the profile after some time. Not long before the moment of the singularity formation, the equation (12) becomes nonapplicable. For correction, it is sometimes sufficient to add into the rhs of the equation (12) the only linear dissipative term $(e^2n/M\sigma)\Delta\mathbf{\Omega}^-$, which takes into account a finite electrical conductivity σ [8]. In this case the equation for the function $\omega^-(q,z,t)$ looks as follows:

$$\omega_t^-+2\omega^-\omega_z^-=\frac{e^2n}{M\sigma}(2q\omega_{qq}^-+4\omega_q^-+\omega_{zz}^-). \tag{15}$$

In order to justify the neglect of dispersive effects, the typical values of ω^- should not be too large $\omega^-<\sim e^2n/2\lambda M\sigma\approx 10e^2n/M\sigma$. With this condition the width of the current sheet will remain several times larger than the dispersive length λ. Otherwise, it is necessary to take into account subsequent terms in the expansion of the Green's function $G_{--}(k)$ on powers of λ^2k^2 (we may neglect the term $1/k^2$ as previously, since $k\gg 1$),

$$G_{--}(k)\approx 1-\lambda^2k^2+(\lambda^2k^2)^2+\cdots, \tag{16}$$

$$\mathcal{H}_\lambda\{\mathbf{0},\mathbf{\Omega}^-\}\approx\frac{1}{2}\int\mathbf{\Omega}^-\cdot(1+\lambda^2\Delta+\cdots)\mathbf{\Omega}^- d\mathbf{r}. \tag{17}$$

Let us consider the axisymmetric flows as (13). It is useful to note that in the absence of dissipation, as follows from Eqs. (7), the dynamics of the functions $\omega^\pm(q,z,t)$ possesses the remarkable structure,

$$\omega_t^\pm+(\delta\mathcal{H}_*/\delta\omega^\pm)_q\omega_z^\pm-(\delta\mathcal{H}_*/\delta\omega^\pm)_z\omega_q^\pm=0, \tag{18}$$

where $\mathcal{H}_*\{\omega^+,\omega^-\}=(1/2\pi)\mathcal{H}_\mu\{\omega^+[\mathbf{e}_z\times\mathbf{r}],\omega^-[\mathbf{e}_z\times\mathbf{r}]\}$. Thus, each of the functions $\omega^\pm(q,z,t)$ is transported by its own, divergence-free in (q,z) plane, two dimensional velocity field, the stream function of which coinciding with the corresponding variational derivative of the Hamiltonian. The same Poisson structure governs the ideal hydrodynamics in Cartesian plane [13].

Using the expression for the Δ operator in (q,z) coordinates,

$$\Delta\{f(q,z)[\mathbf{e}_z\times\mathbf{r}]\}=(2qf_{qq}+4f_q+f_{zz})[\mathbf{e}_z\times\mathbf{r}], \tag{19}$$

we easily obtain the asymptotic expansion (for simplicity, we write ω instead of ω^- in the two following equations)

$$\mathcal{H}_*\{0,\omega\}=\int\omega[q+\lambda^2(2\partial_q q^2\partial_q+q\partial_z^2)+\cdots]\omega\,dq\,dz \tag{20}$$

and the corresponding conservative equation of motion

$$\begin{aligned}\omega_t+2\omega\omega_z+2\lambda^2[&-(2q^2\omega_{qqz}+4q\omega_{qz}+q\omega_{zzz})\omega_q\\&+(8q\omega_{qq}+4\omega_q+\omega_{zz}+2q^2\omega_{qqq}+q\omega_{zzq})\omega_z]=0,\end{aligned} \tag{21}$$

where the nonlinear dispersive terms are explicitly written in the first order on λ^2. The dissipation can be taken into account as in the rhs of the Eq. (15).

In the special case when ω^- is only slowly dependent on the radial coordinate q, but strongly depends on the axial coordinate z, the expansion of $G_{--}(k)$ on the powers of $\lambda^2(k_x^2+k_y^2)$ is appropriate,

$$G_{--}(k)\approx\frac{1}{1+\lambda^2k_z^2}-\frac{\lambda^2(k_x^2+k_y^2)}{(1+\lambda^2k_z^2)^2}+\cdots. \tag{22}$$

Then in the leading order the equation of motion for $\omega^-(z,t)$ becomes nonlocal integral differential,

$$\omega_t^-(z,t)+\omega_z^-(z,t)\lambda^{-1}\int_{-\infty}^{+\infty}\omega^-(\xi,t)e^{-|z-\xi|/\lambda}d\xi=0. \tag{23}$$

For long-scale profiles of ω^- this equation is approximately reproduced by Eq. (14), but in addition, it is able to describe changing of the steeping regime from explosive $|\omega_z^-|_{\max}\sim(t_*-t)^{-1}$ to exponential $|\omega_z^-|_{\max}\sim\exp C(t-t_*)$ after the width of the shock becomes smaller than λ. The exponential growth of the maximum of $|\omega_z^-|$ takes place on the final stage of shock evolution (without dissipation) since the integral operator in Eq. (23) makes the transport velocity for ω^- smooth enough even for a very narrow shock.

Concluding remarks. Analogously, the helical flows can be investigated, with

$$(\Omega^-)^z=\Omega(x\cos Kz+y\sin Kz,y\cos Kz-x\sin Kz,t), \tag{24}$$

$$(\Omega^-)^x=-Ky(\Omega^-)^z,\quad(\Omega^-)^y=Kx(\Omega^-)^z \tag{25}$$

that are space-periodic along z direction with the period $L^z = 2\pi/K$. The general solution of Eq. (12) for this case can also be obtained, since the equation of motion for the function $\Omega(u,v,t)$ is

$$\Omega_t + 2K^2\Omega(v\Omega_u - u\Omega_v) = 0. \tag{26}$$

This equation follows from the Hamiltonian

$$\mathcal{H}_s\{0,\Omega\} = \frac{1}{2}\int \Omega[1 + K^2(u^2+v^2) + \cdots]\Omega\, du\, dv. \tag{27}$$

Each level contour $\Omega(u,v) = W$ rotates with the individual angular velocity $d\theta/dt = -2K^2W$, that is the reason for shock producing. Higher-order corrections to Eq. (26) can be derived similarly to Eqs. (19)–(21). However, in this case it is not possible to include the dissipation into consideration in the framework of single-function description by Eqs. (24)–(25), since magnetic diffusivity destroys helical shapes of the magnetic lines.

If we would like to escape the restriction $k \gg 1$, it would be necessary to deal with the Hall MHD, the Hamiltonian of which is

$$\mathcal{H}^{HMHD}\{\mathbf{\Omega}^+,\mathbf{\Omega}^-\} = \frac{1}{2}\int |\mathbf{\Omega}^-|^2 d\mathbf{r} + \frac{1}{2}\int (\mathbf{\Omega}^+ + \mathbf{\Omega}^-) \times(-\Delta^{-1})(\mathbf{\Omega}^+ + \mathbf{\Omega}^-) d\mathbf{r}. \tag{28}$$

For axisymmetric flows we have

$$\mathcal{H}_*^{HMHD}\{\omega^+,\omega^-\} = \int (\omega^-)^2 q\, dq\, dz + \frac{1}{2}\int (\omega^+ + \omega^-) \times \hat{G}(\omega^+ + \omega^-)\, dq\, dz, \tag{29}$$

where the operator $\hat{G}$ is defined as follows:

$$\hat{G} f(q,z) \equiv \frac{1}{4\pi}\int (qq_1)^{1/4} F\left(\frac{(z-z_1)^2 + 2(q+q_1)}{4(qq_1)^{1/2}}\right) \times f(q_1,z_1)\, dq_1\, dz_1,$$

$$F(A) \equiv \int_0^{2\pi} \frac{\cos\varphi\, d\varphi}{\sqrt{A - \cos\varphi}}. \tag{30}$$

The equations of motion can be written in the form

$$\omega_t^- + (2\omega^- + \Psi_q)\omega_z^- - \Psi_z\omega_q^- = 0, \tag{31}$$

$$\omega_t^+ + \Psi_q\omega_z^+ - \Psi_z\omega_q^+ = 0, \tag{32}$$

$$\Psi = \hat{G}(\omega^+ + \omega^-). \tag{33}$$

Since the nonlocal operator $\hat{G}$ possesses smoothing properties, analogously to the usual "flat" Δ^{-1} operator, the stream function Ψ is smooth enough even where the functions ω^+ and ω^- have infinite gradients. Therefore, the effect of the nonlocality, generally speaking, cannot overcome the tendency towards the breaking of the function ω^- profile, at least with moderate typical values of Ψ. We can suppose that with the initial data concentrated in the region $k \sim 1$, the breaking takes place as the general case in the Hall MHD model. As concerns the transition to the limit of usual MHD, on small $k \ll 1$, and $\omega^- \ll \Psi_q$, $|\omega^+ + \omega^-| \ll |\omega^+|, |\omega^-|$, in this case the question about breaking remains subtle and needs additional investigations.

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[1] D. Biskamp, Phys. Fluids B **5**, 3893 (1993).
[2] H. Politano, A. Pouquet, and P.L. Sulem, Phys. Plasmas **2**, 2931 (1995); H. Politano and A. Pouquet, Phys. Rev. E **52**, 636 (1995); **57**, R21 (1998).
[3] R. Grauer and Ch. Marliani, Phys. Plasmas **5**, 2544 (1998); Phys. Rev. Lett. **84**, 4850 (2000).
[4] W.-C. Müller and D. Biskamp, Phys. Rev. Lett. **84**, 475 (2000).
[5] D. Biskamp, E. Schwarz, and J.F. Drake, Phys. Rev. Lett. **76**, 1264 (1996); Phys. Plasmas **4**, 1002 (1997); D. Biskamp, E. Schwarz, and A. Celani, Phys. Rev. Lett. **81**, 4855 (1998); D. Biskamp *et al.*, Phys. Plasmas **6**, 751 (1999).
[6] M. Ottaviani and F. Porcelli, Phys. Rev. Lett. **71**, 3802 (1993); E. Cafaro *et al.*, *ibid.* **80**, 4430 (1998); D. Grasso *et al.*, *ibid.* **86**, 5051 (2001).
[7] G. Boffetta *et al.*, Phys. Rev. E **59**, 3724 (1999).
[8] A.S. Kingsep, K.V. Chukbar, and V.V. Yan'kov, in *Reviews of Plasma Physics*, edited by B. Kadomtsev (Consultants Bureau, New York, 1990), Vol. 16, p. 243.
[9] A. Fruchtman and Y. Maron, Phys. Fluids B **3**, 1546 (1991); A. Fruchtman, *ibid.* **3**, 1908 (1991); A. Fruchtman and K. Gomberoff, *ibid.* **5**, 2371 (1993); K. Gomberoff and A. Fruchtman, **5**, 2841 (1993).
[10] J. Kalda, Phys. Fluids B **5**, 4327 (1993).
[11] S.I. Vainshtein, S.M. Chitre, and A.V. Olinto, Phys. Rev. E **61**, 4422 (2000).
[12] S.V. Bulanov, F. Pegoraro, and A.S. Sakharov, Phys. Fluids B **4**, 2499 (1992).
[13] V.E. Zakharov and E.A. Kuznetsov, Usp. Fiz. Nauk **167**, 1137 (1997) [Phys. Usp. **40**, 1087 (1997)].
[14] P.J. Morrison, Rev. Mod. Phys. **70**, 467 (1998).
[15] E.A. Kuznetsov and V.P. Ruban, Phys. Rev. E **61**, 831 (2000).
[16] V.P. Ruban, Zh. Éksp. Teor. Fiz. **116**, 563 (1999) [JETP **89**, 299 (1999)]; V.P. Ruban *et al.*, Phys. Rev. E **63**, 056306 (2001); V.P. Ruban, *ibid.* **64**, 036305 (2001).